Novanions and Einstein-Heim unification

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As mentioned in A. D. Fokker's *Time and space, weight and inertia*, Pergamon, [Ref], a book which had enormous influence on me and I read at school, towards the end of his life, in order to incorporate quantum theory, Einstein considered noncommutative gauge theories. It is often considered, for instance as mentioned by Murray Gell-Mann in The *quark and the Jaguar* [Ref] that this later work on unified field theory was not of the same value as his earlier work.

It has recently come to my attention, in a communication with Graham Ennis, that a critique of the Einstein programme had been developed by the German physicist Burkhard Heim, which is close to its spirit, and is successful in predicting many features that can be subject to experiment. This work is remarkable in that it contains a prediction of the value of the fine structure constant, and a formula for the mass-spectrum of fundamental particles. This latter feature is most astonishing since it is the case that conformal field theory and the theory of supergravity have been abandoned as being unable to generate a sufficiently profuse elementary particle spectrum [Ref].

The work of Burkhard Heim connects general relativity with magnetic effects. It is interesting that the last experiments of Faraday were intended to investigate such a connection (this paper is omitted from his collected works). There has been work by Clifford Will on the experimental basis of general relativity and theories which differ from it.

The theory of superstrings (Witten, Schwartz and Green, [Ref]) is sometimes criticised (*Not even wrong*, [Ref]) as not supplying sufficient contact with experiment, and of being an ansatz of relativity and quantum mechanics with no independent inherent theoretical justification. However, it *is* the case that gravity affects selection rules in quantum theory (Steven Weinberg, *The quantum theory of fields* [Ref]).

The objective of this little note is to connect Heim's work with the theory of novanions. The mathematical basis of novanion theory is given in Jim H. Adams, *Superexponential algebra*, volume I, chapter V, www.jimhadams.com, 2015. The connection of quaternions with special relativity, which are embedded in novanions, is given in the early work by the same author, *Vector calculus*, although novanions were not discovered until later. An account of the novanion theory where the point of view is that the theory describes strings is given in Jim H. Adams, *Electroweak chirality*, *n-novanions and the heterotic string*. Our point of view is now a little different from the one given in that paper.

A good account of Riemannian geometry is given in the exposition of Pereleman's work on the Poincaré hypothesis [Ref], and we will incorporate some features of that account here.

As was discovered by the mathematician Hassler Whitney [Ref], Riemannian geometries of finite local dimension may be embedded in flat manifolds. In the case of general relativity the local metric (this generalises the Pythagoras theorem to relativistic line elements) is given by

$$
\begin{aligned}
ds^2 = {} & g_{00}dx^0dx^0 + g_{01}dx^0dx^1 + g_{10}dx^1dx^0 + g_{02}dx^0dx^2 + g_{20}dx^2dx^0 \\
& + g_{03}dx^0dx^3 + g_{30}dx^3dx^0 \\
& + g_{11}dx^1dx^1 + g_{12}dx^1dx^2 + g_{21}dx^2dx^1 + g_{13}dx^1dx^3 + g_{31}dx^3dx^1 \\
& + g_{22}dx^2dx^2 + g_{23}dx^2dx^3 + g_{32}dx^3dx^2 \\
& + g_{33}dx^3dx^3, \qquad (1)
\end{aligned}
$$

where the gauges g_{ij} are symmetric:

$$g_{ij} = g_{ji}.$$

This symmetry means the 16 components are reduced to 10:

$$
\begin{aligned}
& g_{00}, g_{01}, g_{02}, g_{03} \\
& g_{11}, g_{12}, g_{13} \\
& g_{22}, g_{23} \\
& g_{33}.
\end{aligned}
$$

This means four dimensional general relativity may be embedded in flat space-time with 10 dimensions, since this corresponds to the 10 degrees of freedom given by the 10 independent gauge components. Our point of view here is that the 10-dimensional flat structure is that of the 10-novanions.

It is also the case that under a perturbation of the gauge coefficients, so that they transform continuously, then this also appears in the novanionic representation as continuous deformations in the embedded manifold, and so in this sense the representation is ‘good’.

10-novanions have properties which differ from most mathematical structures considered in physics, namely that they are not associative (although Lie algebras are not associative). An identical way of saying this is that the position of brackets matters when considering the evaluation of novanion expressions containing multiplication.

A further aspect is that when the scalar part is zero, and only in these circumstances, novanions do not conserve number. We identify the scalar part of a novanion with time, for which we have only one component. All other components are novanionic-imaginary, noncommutative and nonassociative with other such imaginary components, but commutative and associative with the scalar temporal part.

The Heim theory contains discrete space-time. We mention a possible feature of novanionic structures, that Wedderburn’s little theorem may not hold, which we now investigate. This states that finite division rings are commutative. Novanions are division algebras except at time t = 0, and in the associative case division algebras correspond to the quaternions which are embedded within novanions, so there are no finite versions of quaternions according to this theorem. However, although Wedderburn’s little theorem does not necessarily apply to nonassociative structures, it does imply in this case that it can contain no finite structures which are quaternionic, even though it can contain quaternions with continuous values.

A possible solution to this problem is that we should deal with novanionic rings, that is, we have addition and subtraction in the theory, and multiplication, but because the theory is built up from finite elements, there is no inherent division. A property of the novanions is retained by novanion rings that it is only at time $t = 0$ that two nonzero novanions can multiply together to give zero, or vice versa, that zero can be the product of two nonzero novanions.

It is now not a contradiction that a novanionic ring can be built up from finite elements, since the proof of a Wedderburn type theorem, which in one implementation uses the fact that by Lagrange's four-square theorem, every number can be represented by four squares, so that in finite congruence arithmetic mod m, m may be represented as the sum of four squares, and a quarternion inverse results in dividing by a quaternion with its quaternionic conjugate, the denominator of which is the sum of four squares, and if this has the value m we are dividing in this arithmetic by m which is equivalent to dividing by zero, and this contradiction carries over to novanions, where this is impossible if the coefficients of the novanion basis elements are real, because novanion division algebras have the same type of inverse structure as quaternions and quaternions are embedded within them.

The Heim theory treats the general relativistic metric as having real components, with time as imaginary and introduces two further imaginary dimensions. We have to prove that for 10-novanions, its embedded surface structure describing general relativity allows orthogonal components with imaginary novanionic values, that is, components at right angles to the general relativistic surface. From a change of basis of the coordinate system, which is possible for novanions, this is possible, and the imaginary components are novanionic imaginary.

However, for novanions the time coordinate is real and all the space components are imaginary. We circumvent this difficulty by specifying that the metric given by (1) is given by purely arbitrary values of the g_{ij}. It is then possible to split each g_{ij} into a symmetric part with $g_{ij} = g_{ji}$ and an antisymmetric part with $g_{ij} = -g_{ji}$. The Hermitian structure of the metric in Heim theory is now transferred to the properties of a decomposition of the gauges into symmetric and antisymmetric parts. Since the novanionic structure contains noncommutative basis elements, multiplicative structures on the gauges are transferred to the multiplicative noncommutative structures on the novanionic basis elements, further constraining the type of embedding. Thus it appears that novanions may be used to describe the six dimensional Heim theory.