Wallace Adams, Graham Ennis, Doly García and Tim Gibbs

The optimal arrangement of reflectors in solar panel tracker arrays

Wallace H. Adams
Mathematical researcher
www.jimhadams.com

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by Wallace Adams, Graham Ennis, Doly García and Tim Gibbs

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Abstract: *We consider plane mirrors surrounding a solar panel array in which the sun's incident radiation is at right angles to the panel, as it would if the panel were configured to track the sun. A specified constraint is that the incident radiation on the solar panel with reflectors is 3 times that of the PV panel alone, ignoring inefficiencies of the reflecting film. Other magnifications are discussed. We find the optimum configuration of the mirrors is at 55.7° to the horizontal axis defined by the* PV (*photovoltaic*) *panel, and the length of this edge reflector is* 0.65 *of the length of the panel, provided the radiation from each reflector covers all of it. We also consider corner mirrors with the same objective. The PV panel considered is square.*

The problem: Consider light shining vertically on a PV panel surrounded by plane mirrors at an angle θ to the horizontal. What is the optimum value of θ so that the entirety of the panel is illuminated?

A PV array with mirrors,
not of the optimal design
calculated in this paper.

The detailed mathematical solution: We depict the angle, θ radians of the mirror compared with the horizontal panel baseline, the length of the panel, n, the length of the mirror, m/cos θ, and the angle ϕ to the vertical with respect to the mirror with which the sun's rays hit the mirror.

solar panel
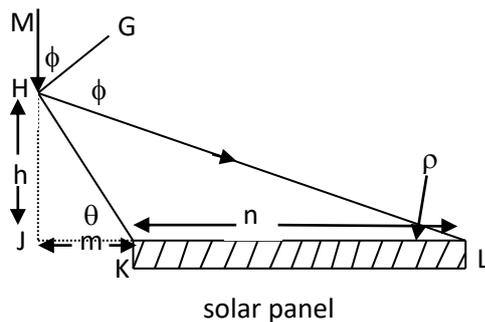
We will assume that at the furthest edge of the mirror a light ray is reflected to hit the far end of the PV panel.

For triangle HKL, the interior angles sum to π radians so, since $G\hat{H}K$ is a right angle and JKL is a straight line

$\rho + ((\pi/2) - \phi) + (\pi - \theta) = \pi,$

or

$\phi = (\pi/2) + \rho - \theta.$ (1)

MHJ is a straight line and the interior angles of triangle HJL sum to π radians, so

$$2\phi + ((\pi/2) - \rho) = \pi,$$

or

$$2\phi = (\pi/2) + \rho. \tag{2}$$

Eliminating ϕ from (1) and (2) gives

$$(\pi/2) + \rho = \pi + 2\rho - 2\theta,$$

or

$$\rho = 2\theta - (\pi/2). \tag{3}$$

It is clear from the previous diagram that in this equation $\pi/2 > \rho > 0$.
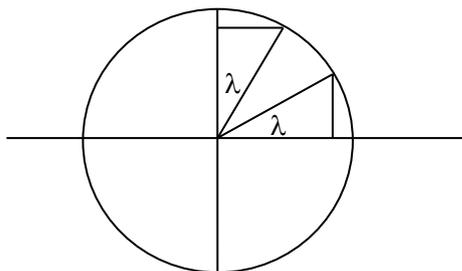
In triangle HJL

$$h/(n + m) = \tan \rho, \tag{4}$$

and in triangle HJK

$$h/m = \tan \theta. \tag{5}$$

Thus on eliminating h from (4) and (5)

$$m \tan \theta/(n + m) = \tan (2\theta - (\pi/2)). \tag{6}$$

Now in the diagram

$$\sin((\pi/2) - \lambda) = \cos \lambda = \cos -\lambda$$
$$\cos((\pi/2) - \lambda) = \sin \lambda = -(\sin -\lambda),$$

so putting $\lambda = 2\theta$

$$(n + m) = -m (\tan \theta)(\tan 2\theta). \tag{7}$$

But

$$\sin^2\theta + \cos^2\theta = 1, \tag{8}$$

and

$$\cos 2\theta = \cos^2\theta - \sin^2\theta, \tag{9}$$

so that from (8), (9) gives

$$\cos 2\theta = 2\cos^2\theta - 1 \tag{10}$$

whereas

$$\sin 2\theta = 2 \sin\theta \cos\theta. \tag{11}$$
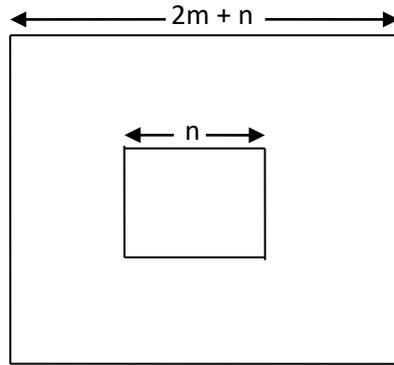
Thus on putting $\tan = \sin/\cos$, (7) becomes

$$(n + m) = -m (\sin \theta/\cos \theta)(2\sin \theta \cos \theta)/(2\cos^2\theta - 1),$$

so

$$[2(n + m) - 2m] \cos^2\theta = -2m + n + m,$$
$$\cos^2\theta = (n - m)/2n. \tag{12}$$

Our specification is that the total output of the PV array with mirrors should not exceed k times the output of the PV array on its own. We are particularly interested in magnification k = 3.

The output of the PV array is 1 unit, so that the output from the mirrors should be (k − 1) units, in order for this to sum to k units. Thus with perfect reflectivity

$$(2m + n)^2 - n^2 = (k - 1)n^2,$$

$$2m + n = (\sqrt{k})n$$

$$m = [(\sqrt{k}) - 1]n/2, \tag{13}$$

so that

$$m = 0.207n \ (k = 2), \text{ or } 0.366n \ (k = 3). \tag{14}$$

Then from (12) in (13)

$$\cos^2\theta = (3 - \sqrt{k})/4$$

$$= 0.39645 \ (k = 2), \text{ or } 0.3170 \ (k = 3)$$

$$\cos\theta = 0.62963 \ (k = 2), \text{ or } 0.5630 \ (k = 3)$$

giving

$$\theta = 50.97° \ (k = 2), \text{ or } 55.7° \ (k = 3). \tag{15}$$

The length of the mirror is $m/\cos\theta$ = 0.33 (k = 2), or 0.65 (k = 3) of the length of the panel.

There is no need to extend the edge reflectors so that they are not rectangles.

If we include 4 mirror diamond shapes at the corners, tipped so that the corner of each mirror meets the corner of the array, then the diagonal for a square array is $(\sqrt{2})n$, so that if a corner mirror is a diamond shape meeting with edge mirrors of length 0.33n (k = 2) or 0.65 (k = 3) and is again aligned at 51° (k = 2), or 55.7° (k = 3) then given the linear proportionality of the radiation on the array, these 4 corner mirrors spread sunlight over the entire diagonal to the other corner of the PV part of array. However, a calculation shows that a corner mirror, if connected by a diagonal to the edge mirrors, is not at an angle 51° (k = 2) or 55.7° (k = 3) to the horizontal, so the corner mirrors should be set back at the lesser angle of 51°(k = 2) or 55.7° (k = 3). The width of this corner mirror diagonal, which we will assume is approximately the same in the 51°(k = 2) or 55.7° (k = 3) configuration, would be 0.29 units (k = 2) or 0.52 units (k = 3). On the other hand the height of the second diagonal at right angles to this diagonal is of $0.33(\sqrt{2})$ = 0.47 units (k = 2) or $0.65(\sqrt{2})$ = 0.92 units (k = 3) in order for the solar radiation at its tip to meet the other corner of the PV part.
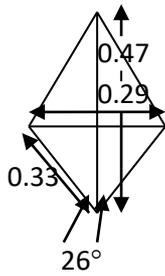
The corner diamond shaped mirror, magnification k = 2
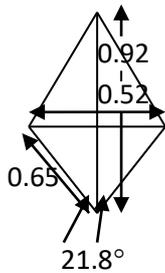
The corner diamond shaped mirror, magnification k = 3

Note that the diagonal width of a corner mirror does not completely illuminate the PV array. The area of the corner mirror is (1/2)(0.47)(0.29) = 0.07 square units (k = 2), or (1/2)(0.92)(0.52) = 0.24 square units (k = 3) there are 4 of them and they are inclined at an angle 51° (k = 2), or 55.7° (k = 3) to the horizontal, thus the power output from the 4 corner mirrors is 0.07×4×0.6296348 ≈ 0.17 of the PV array (k = 2), or 0.24×4×0.5630 ≈ 0.54 of the array (k = 3).

We will assume the mirror film has properties similar to 3M™ Cool Mirror Film 330, with an average reflectance at normal incidence of 89% or greater. This reflects light that is useful for PV modules and transmits infrared light. When used in low-x concentrated photovoltaic systems, it can be used to increase the amount of usable light on the module while limiting the amount of heat-generating infrared light on the module. At normal incidence angles, it has low visible reflectance, which limits blinding reflections.

This means that the total configuration can be enhanced to reach 1.0 + 0.89 ≈ 1.8 times the power output of the array alone (k = 2), or 2.8 (k = 3). This is suitable for the maximum tripling capacity of the PV array, also given that the irradiation from the corner mirrors is more concentrated closer to the PV centre.