Elementary methods in number theory

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Jim H. Adams is a researcher in the concepts of mathematics and their explicit representation. This work is a guide to some of my early ideas. It was assembled with light editing out of the eBook *Exponential Factorisation theorems*, of 2009, and three papers on my website, two on Fermat's last theorem, and one on Beal's conjecture. It is also an encouragement to the aspiring mathematician to get involved in mathematical research at an early stage, and continually.

After a career in IT, Jim joined New Music Brighton as a composer and performer of his own works. He has been extensively involved in mathematical research.

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Foreword

In this outline of our conceptual work, we describe explorations, which have been common mathematical currency for over two centuries, of the mathematical landscape concerning the theory of whole numbers. It is a review based on elementary methods, where the game is one of not using complex numbers.

In chapter 1 discussing global field theorems for exponential powers, section 2 introduces an exponential notation. Section 3 restates foundational rules for real exponentiation, providing proofs for the basic *'binomial exponent'* and *'geometric exponent'* theorems. Then section 4 continues with new cyclotomic variants of the *'Fermat subtraction', 'Fermat addition'* and *'linear combination'* factorisation theorems, the latter being a formula that is a linear combination of the previous two. Finally section 5 develops in many variables the linear combination factorisation theorem.

Prime number, factorisation and divisibility theorems are discussed in chapter 2. In particular, we obtain primality conditions not dealt with in most textbooks, e.g. for '*generalised Fermat'* numbers, for positive natural numbers γ or $\delta > 1$ and p, we prove that no number of the form $\gamma^p + \delta^p$ is prime, except for the possibilities p = 1 or p a power of 2. For *'generalised Mersenne'* numbers, no representations of primes are of the form $\gamma^p - \delta^p$, except for the possibilities p = 1, or $\delta = (\gamma - 1)$ and p prime. We also discuss a linear combination of powers prime number theorem and continue with a discussion on factorisation of pth powers $\gamma^p \pm \delta^p$. This chapter also discusses extensions of *Fermat's little theorem*, including theorems connected with *reciprocity*. We make some remarks on *Fermat's last theorem*.

Chapter 3 analyses using elliptic curves the number of solutions mod 4 of differences and sums of pth and different powers.

Generalised Quadronacci numbers are discussed in chapter 4 – each number is the sum of the previous four, and generalised other such sequences – of Fibonacci, Lucas and Tribonacci numbers.

In chapter 5, we discuss approaches to Fermat's last theorem.

Chapter 6 provides some exploration by elementary methods of Beal's conjecture – which is a generalisation of Fermat's last theorem.

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