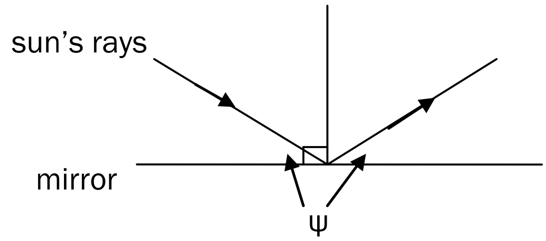
2.7. Additional output using reflectors.

We now look at various set-ups for reflectors and their resulting output.

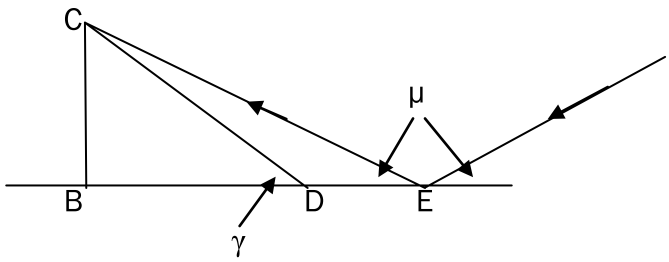
2.7.1. Angles of incidence and reflection.

A standard result we will use in the diagrams depicting light rays is that the angle of incidence on a mirror equals the angle of reflection, ψ, shown below.



2.7.2. The main reflector set-up.

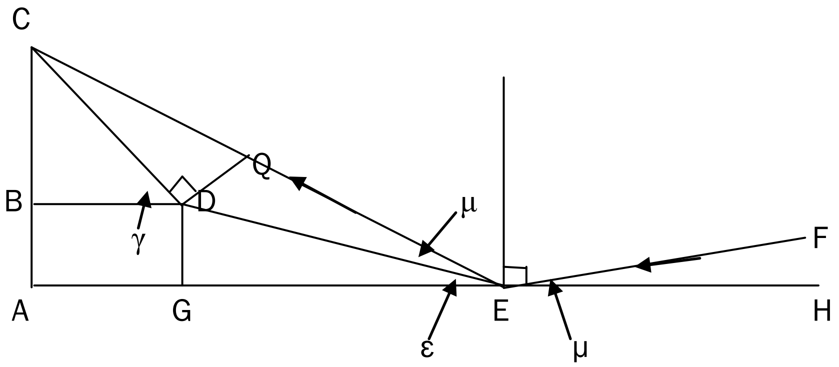
We will first look at the case of direct sunlight on a panel CD at an angle γ to the horizontal where a reflector is placed horizontally next to it.



Suppose the sun's rays are at angle μ to the horizontal. We have seen that the sun's rays are at right angles to a panel at month m when the slope of the panel is 43.7°. This means the sun's rays meet the horizontal at

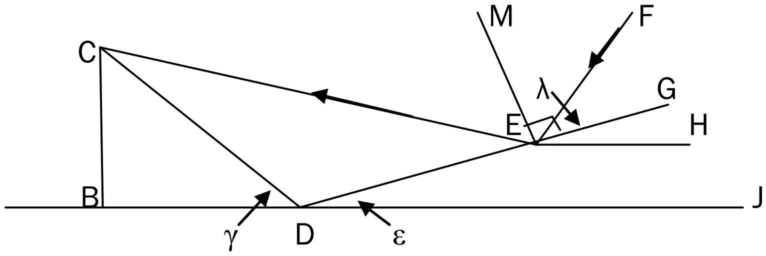
$\mu = 90° - 43.7° = 46.3°$.

For angle $\gamma = 28.4°$, this is less than μ, and so for these angles a computation gives the length DE of the flat reflector unphysically as negative! This means we might consider whether the following set-up is possible instead.



The panel at CD is now elevated by a height BA = DG above the flat roof, and the reflector along DE touches the flat roof at E. The edge ray FE continuing to C is now at an angle CEG = $\mu + \varepsilon$ to the horizontal, and since angle $\mu = 46.3°$ and angle $\gamma = 28.4°$, so $\gamma < \mu$, this is even worse than the horizontal case previously discussed.

We are therefore led to consider the case when the reflector is at an angle to the horizontal shown below.



The panel CD now adjoins the reflector along DE (or there is a space, for rainwater). The panel angle to the horizontal is γ, the reflector angle is ε, EH is a horizontal line and the light ray FE meeting the reflector at E makes an angle of λ with the continuation of DE to DG. Thus

angle EDJ = ε = angle GEH,

so since

angle FEG = λ = angle GEH,

we have the angle of the light ray to the horizontal

angle FEH = $\mu = \lambda + \varepsilon$.

If E is the extremity of the reflector, and a light ray FE is reflected to the end of the panel at C, since DCE is a triangle whose angles sum to 180°,

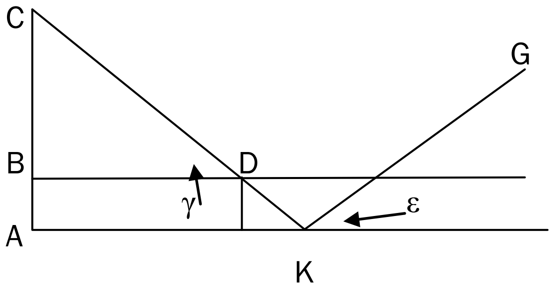
angle DCE + angle CED + angle EDC = 180°,

angle DCE + λ + (180° – γ – ε) = 180°,

so

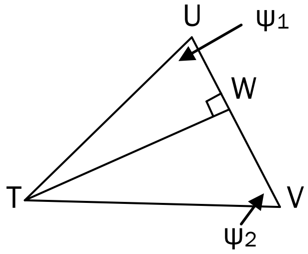
angle DCE = $\gamma + \varepsilon - \lambda$.

We are recommending that to allow rainwater to escape, the panel CD should be elevated to a height BA. Then



an imaginary continuation of CD meets the floor at K and the reflector is now along KG.

We now use the sine rule, whose proof we first provide. Let TUV be the triangle shown below



We have

$\sin \psi_1 = TW/TU$,

$\sin \psi_2 = TW/TV$,

so

$TW = TU \sin \psi_1 = TV \sin \psi_2$,

giving

$\sin \psi_1/TV = \sin \psi_2/TU$.

Consequently

sin (angle DCE)/DE = sin(angle CED)/CD.

Hence

$DE = CD \sin (\gamma + \varepsilon - \lambda)/\sin \lambda$.

If $\gamma = 30°$, say, where $\mu = 46.3° = \varepsilon + \lambda$, then

$DE = CD \sin(30° + 2\varepsilon - 46.3°)/\sin(46.3° - \varepsilon)$.

Thus a necessary requirement for DE to be positive is that

$2\varepsilon > 16.3°$

so

$46.3° > \varepsilon > 8.7°$,

where ε is the slope of the reflector.

For instance, if we chose $\varepsilon = 30°$ then

$DE = CD \sin 43.7°/\sin 16.3° = 2.143$ CD.

However, we have seen at the winter solstice that $\mu = \lambda + \varepsilon = 15.8°$. If we do not want to block out the sun's rays from the panel at any time of year, since this would introduce shadow and for some systems would inhibit all output from the panel array, then at a minimum value of λ, $\lambda = 0$, we must have ε at most as, say, 15°, which we will use in calculations, although 15.5° might squeeze a little more. If we assume the 15° value is the optimum, then we can extend DE in terms of CD, say DE = 2.5 CD.

It is advised that the value of the length of the reflector be larger than this, so that for smaller values of μ (winter!), the reflector continues to beam over the entire panel. Also panels in a rank in front of the panel should not obstruct the view of the reflector. Let us assume that DE = 3CD. Since for small angles $\sin(\psi)$ = angle ψ radians, this will enhance our calculation by approximately 1.2 times from DE = 2.5 CD.

We have seen that angle FEH is

$\varepsilon + \lambda = \mu = 46.3°$

so that with $\varepsilon = 15°$

$\lambda = 21.3°$.

Also we showed the angle of the reflected light ray to the panel is

angle DCE $= \gamma + \varepsilon - \lambda = 30° + 15° - 21.3° = 23.7°$,

so the component of reflected sunlight at right angles to the panel is

$\sin(23.7°) = 0.402$.

At $\varepsilon = 15°$, $\mu = 46.3°$ and the light ray at E just reaches C. After this, as the angle λ increases into summer, E is effectively displaced and moves towards D. When ED = 0, then μ effectively satisfies

$30 + 2\varepsilon - \mu = 0$,

$\mu = 60°$.

Since at the summer solstice $\mu = 62.6°$, and at the winter solstice $\mu = 15.8°$ there is no contribution from the reflector for

$$100 \times 2.6/(62.6 - 15.8) = 100 \times 2.6/46.8 = 5.6\% \text{ of the year.}$$

We now do a fairly sophisticated computation of the efficiency of this configuration. With

$$3CD = CD \sin(30° + 15° - \lambda)/\sin(\lambda),$$
$$3\sin(\lambda) = \sin(45° - \lambda) = \cos(45°)\sin(-\lambda) + \sin(45°\cos(-\lambda),$$
$$= (-\sin(\lambda) + \cos(\lambda))/\sqrt{2},$$
$$\tan(\lambda) = 1/(3\sqrt{2} + 1) = 1/5.2426 = 0.19,$$

so

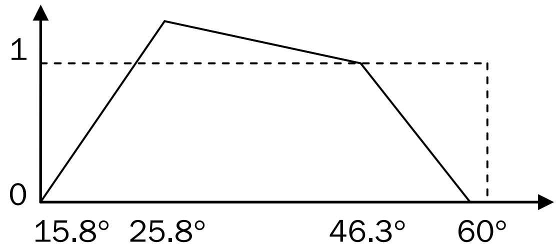
$$\lambda = 10.8°.$$

Thus

$$\mu = \varepsilon + \lambda = 25.8°,$$

and angle DCE at $\mu = 25.8°$ is $30° + 15° - 10.8° = 34.2°$, so that the component of reflected sunlight to the panel is $\sin(34.2°) = 0.5621$.

So the illumination at right angles to the panel is at 0 units for $\mu = 15.8°$, increases to $0.5621/0.402 = 1.4$ units for $\mu = 25.8°$, decreases to 1 unit for $\mu = 46.3°$ and then decreases to 0 units for $\mu = 60°$. Using the result that the area of a triangle is given by ½ base × height,



the area of the trapezium is

$$½(25.8° - 15.8°) + (1 + 0.4/2)(46.3° - 25.8°) + ½\,(60° - 46.3°) \text{ units}$$
$$= 5 + 1.2 \times 20.5 + (13.7/2) = 5 + 24.6 + 6.85 = 36.45 \text{ units},$$

and the area of the rectangle is $(62.5° - 15.8°) = 46.7$ units, so the component of the reflected sunlight is reduced by

$$0.402 \times 36.45/46.7 = 0.314.$$

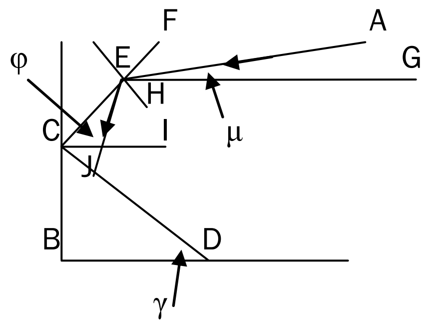
As a second consideration, we will investigate what happens under indirect illumination, where we will consider the radiation for the above ε, which now must be the case. Since we have seen that $\mu = \varepsilon + \lambda = 15° + \lambda$, say, which means indirect radiation varies from 0° to $\lambda = \mu - \varepsilon = 30.3°$, its proportion of the sky is $30.3/180 = 0.168$, or 16.8%.

Relative to the component of direct sunlight at 1, the component of indirect light is 1.11. Thus over the whole year, the total percentage increase in using reflectors taking account of both direct and indirect illumination is

$$100 \times [1 \times 0.314 + 1.11 \times 0.168]/[1 + 1.11] = 23.7\%.$$

2.7.3. Illuminating the panel from above.

We will look at a reflector suitable for a back array of arrays in ranks, so that there is no shadow from the reflector for arrays in other ranks, as would be arranged from the diagram



We note that in this set-up, part of the indirect illumination on the panel is blocked off, so we will need to compute this.

The panel is along CD at angle γ to the horizontal, the reflector along CF is at angle φ to the horizontal, and the sun's rays, striking the reflector at E are at an angle μ to the horizontal.

Now

angle ICJ = angle DBC = γ

angle ECI = φ

so

angle ECJ = angle ECI + angle ICJ = $\varphi + \gamma$,

whereas

angle FEG = angle ECI = φ,

so

angle AEF = angle FEG – angle AEG = $\varphi - \mu$.

What is angle EJC, the angle at which the reflected light ray hits the panel CD? From the property of the interior angles of a triangle

angle EJC + angle JCE + angle CEJ = 180°,

where we have seen

angle JCE = $\varphi + \gamma$,

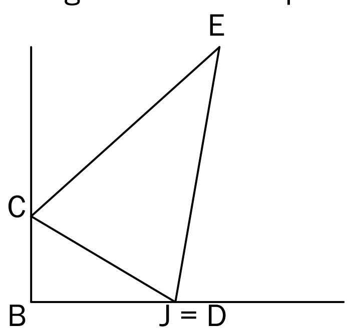
angle CEJ = angle FEA = $\varphi - \mu$.

Thus

angle EJC = $180° - \varphi - \gamma - \varphi + \mu$

$= 180° - 2\varphi - \gamma + \mu$.

For the length of the corresponding lines, if CJ is extended to CD as in the diagram below



angle CEJ = $\varphi - \mu$,

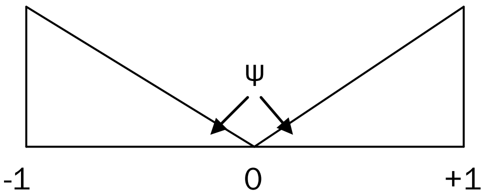
angle EJC = $180° - 2\varphi - \gamma + \mu$,

and by the sine rule

[sin(angle CEJ)]/CJ = [sin(angle CJE)]/CE, so CE, the length of the reflector to cover the whole of the panel CJ = CD, is

$$CE = CD[\sin(180° - 2\varphi - \gamma + \mu]/\sin(\varphi - \mu),$$

and since for an arbitrary angle ψ given in the symmetrical diagram below



$$\sin \psi = \sin (180° - \psi),$$

we have the length of the reflector

$$CE = CD[\sin(2\varphi + \gamma - \mu]/\sin(\varphi - \mu).$$

For example, if CE = CD, then

$$180° - 2\varphi - \gamma + \mu = \varphi - \mu,$$

$$3\varphi = 180° - \gamma + 2\mu,$$

or

$$\varphi = 60° + (2\mu - \gamma)/3.$$

At the mid month m = 18th April we are taking $\mu = 46.3°$, so this gives the slope of the reflector

$$\varphi = 81°.$$

We recommend $\varphi = 82°$ and extending CE to 2.3 CD, so that the panel continues to be covered by reflected sunlight in summer. For reasons given at the end of this section, it is not desirable that CE be too large, which is why we have increased φ by 1°. Under the conditions of the summer solstice we now have

$$CE = CD \sin(180° - 164° - 30° + 62.6°)/\sin(82° - 62.6°)$$

$$= CD \sin(46.8°)/\sin(19.4°) = 2.26 \text{ CD}.$$

The direct component of solar radiation for $\varphi = 81°$ is at right angles to the panel by the amount

$$\sin(\text{angle EJC}) = \sin(180° - 2\varphi - \gamma + \mu) = \sin(18° - 30° + 46.3°)$$

$$= \sin(34.3°) = 0.564.$$

The indirect component varies over

$$\text{angle FEG} = \varphi = 81°,$$

where the total component for unreflected indirect radiation is 180°.

However, for indirect illumination we are also cutting off $180° - \varphi$ of the angle at C. This means that with the reflector in this position, the indirect radiation covers

$$\varphi - (180° - \varphi) = 2\varphi - 180° = -18°,$$

so the reduction of indirect illumination is 18/180 = 0.1.

Thus if we sum the direct components for the reflector at 15° with this top reflector, together with the positive and negative indirect components, we get an increase of

$$100 \times [1 \times (0.314 + 0.564) + 1.11 \times (0.168 - 0.1)]/[1 + 1.11]$$

$$= 100 \times [0.878 + 0.075]/2.11 = 100 \times 0.953/2.11$$

$$= 45\%.$$

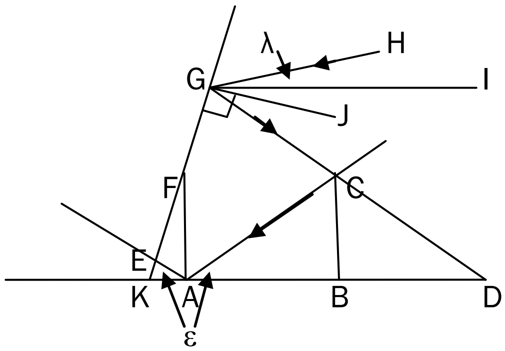
We must bear in mind that the top reflector panels are not available in this configuration to ranks of panels in front of them.

Particles of dust degrade the performance of solar panels. Normal rainwater will wash the panel and thus maintain its effectiveness. The set-up we are discussing will shield part of the panel from rainwater in some circumstances.

Further, the high exposure of the reflector to wind makes it necessary to secure it firmly (no part of the structure to do this should cast a shadow on the panel). There could be a destructive failure of the support structure for the reflector during a gale. It is not recommended that this option is tried unless the structure is stable in a gale, or a movable reflector angle is made to offer minimal wind resistance in such a circumstance.

2.7.4. Illuminating the panel from below.

Why do a calculation on the set-up of panels illuminated from below? The underside of the panel can be illuminated perfectly at midday, but at other times a shadow would be cast due to the struts holding up the panel. This means no additional output would be produced for the time the underside contained partial shadow. However, it is possible to extend the width of the structure on which the panel rests, so its edges meet the floor some distance away from the panel, and this would increase the acceptability of this method summed over a longer period of the day.



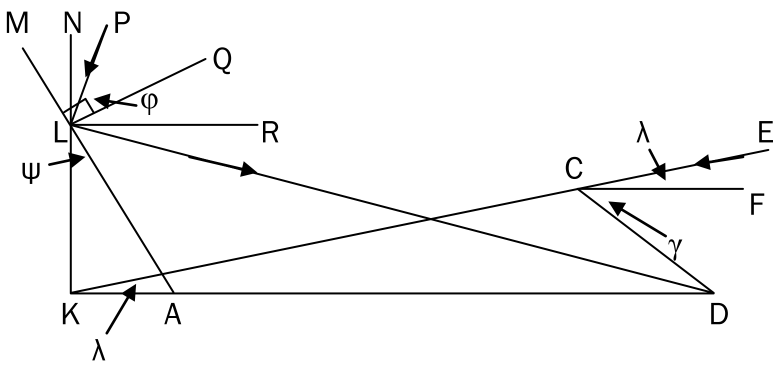
If in the diagram above CD is the panel and EG the reflector, at the winter solstice, a light ray along CA will meet the horizontal AB at an angle ε. At the summer solstice, a light ray HG if at angle λ to the horizontal hits the panel after reflection at C.

This arrangement is inconsistent if angle GKA $\leq 90°$, because $\lambda = 62.6°$, so

$2\lambda >$ angle HGK $= 2\lambda +$ angle IGJ.

Thus if angle GKA $\leq 90°$, we must exclude the illumination for part of the year. There are two extremes: exclude illumination for a period around the summer solstice, or exclude illumination around the winter solstice. However, we have just seen that there can be no illumination below the panel at the summer solstice if angle GKA $\leq 90°$.

So let us look at the case angle GKA > 90° above, corresponding to angle LAD below.



Let the panel be along CD with slope γ. At the winter solstice a light ray at angle $\lambda = 15.8°$ goes from C on the panel to K on the flat roof. KN is at right angles to KD. The reflector is aligned along AM at angle ψ to the vertical. KN meets AM at L. QL is at right angles to AL.

In general a light ray hits the reflector along PL and meets the base of the panel at D.

angle PLQ = φ = angle QLD.

Angle NLR is a right angle.

To find the separation length KD in terms of the length of the panel CD we note

angle KCD = 180° – angle ECD = 180° – λ – γ,

so that by the sine rule

sin(180° – λ – γ)/KD = sin angle CKD/CD,

or

KD = CD sin(λ + γ)/sin(λ) = CD sin 45.8°/sin 15.8° = CD × 0.7169/0.2723
= 2.633 CD.

At the winter solstice

angle PLR = λ,

so

angle PLD = 2φ = angle RLD + angle PLR,

giving

angle RLD = 2φ – λ.

We want to obtain the value of φ at the winter solstice. Note that

angle QLR = ψ

and that

angle QLR + angle RLD = φ,

which means

$\psi + 2\varphi - \lambda = \varphi$,

or

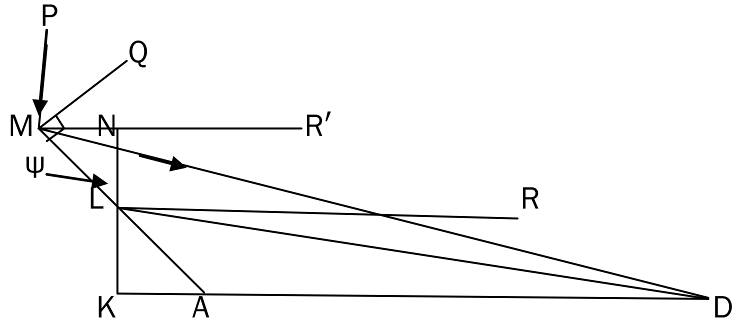
$\varphi = 15.8° - \psi$.

We will fix this value of L as the bottom edge of the reflector. For higher values of λ up to $\mu = 62.6°$ at the summer solstice, the reflection will be higher up along the reflector at LM. At the summer solstice, angle PLR is a maximum

angle PLR = μ = 62.6°.

The bottom part of the reflector ML at L will now illuminate part of the floor KD, and this is acceptable. We would like a light ray from the top edge, M, of the reflector to hit the top of the panel CD at C. This will then give the length ML of the reflector to illuminate the underside of the panel throughout the year.

We first ask, and this will allow us to exclude this option, what value of L, now the effective bottom part of the reflector at the summer solstice, displaced to M is needed so that a light ray from M will hit the bottom of the PV panel at D?



We have (the reader may wish to skip the calculation and go to the conclusion at the end of this section)

$$\text{angle NML} = 90° - \psi,$$
$$\text{angle PMN} = \mu,$$

so

$$\text{angle PML} = \text{angle PMN} + \text{angle NML} = \mu + 90° - \psi,$$

giving

$$\text{angle PMQ} = \text{angle PML} - 90° = \mu - \psi = \text{angle QMD},$$

thus

$$\text{angle PMD} = 2 \times \text{angle PMQ} = 2(\mu - \psi) = 133.2° - 2\psi \text{ .}$$

We also have

$$LK = LA \cos(\psi),$$
$$LK/KD = \tan(\text{angle RLD}) = \tan(2\varphi - \lambda) = \tan(15.8° - 2\psi),$$

so

$$LA \cos(\psi) = KD \tan(15.8° - 2\psi).$$

Then by a constant of proportionality β

$$\sin(\text{angle RLD}) = \beta LA,$$
$$\sin(\text{angle R'MD}) = \beta MA$$
$$(MA - LA)/MA = [\sin(133.2° - 2\psi) - \sin(2\varphi - 15.8°)]/\sin(133.2° - 2\psi),$$

but

$$LA = 2.64\ CD \tan(15.8° - 2\psi)/\cos(\psi),$$

so

$$MA - 2.64\ CD \tan(15.8° - 2\psi)/\cos(\psi)$$
$$= MA[\sin(133.2° - 2\psi) - \sin(2\varphi - 15.8°)]/\sin(133.2° - 2\psi),$$
$$MA = 2.64\ CD\ [\sin(133.2° - 2\psi)\cos(15.8° - 2\psi)]/\cos(\psi).$$

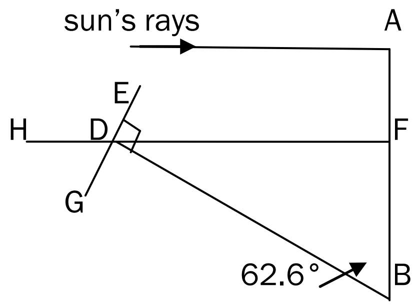
The value of MA and LA vary widely for ψ between 6° and 7°, which are just the sort of values that we should be looking for. Thus this set-up is too dependent on the exact value of ψ and is not appropriate.

2.7.5. Possibly warranty compliant solutions.

The warranty for a PV panel will specify that the conditions under which it operates must not exceed its design characteristics. However, it may be possible to assume that if the panel operates under conditions equivalent to those at the equator, then it does not exceed its warranty requirements. Alternatively, if the PV panel is designed for a certain minimum latitude on the Earth's surface, then under either of these conditions, it might be possible to boost the irradiance of the panel to just below its stipulated maximum, without infringing warranty requirements.

If this condition holds on the warranty, then we can calculate the maximum boost from a reflector to a solar panel which maintains it within the design characteristics for normal operation. Thus it may be of interest to calculate the relative irradiance at the equator and that for the latitude at which the panel is situated.

Say at the equator the incident solar radiation has value 1, and its direct and scattered plus cloud components are in the same ratio as considered previously. Then at the summer solstice for Brighton and Hove at D



angle HDG = 62.6°,

so this means the vertical component of the sun's rays at D are

$\sin(62.6°) = 0.8878$

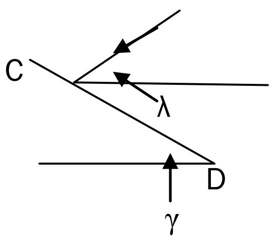
so we can boost output and stay within the warranty at that time by at least

$100 \times (1 - 0.888) = 11.2\%$.

2.7.6. Maximum output.

The assumption we have made up to now is that we wish to maximise the output from the winter solstice up to 18th April, which should equal the output from 18th April up to the summer solstice. We now recalculate the optimum slope of the solar panel with the horizontal floor under the assumption that the output from both direct and indirect solar radiation is optimised over the whole year. Having done this, we recalibrate the results under the new assumption and compare them with the old. We should note, however, that the old calculations reduce the output at the summer end and increase them at the winter end compared with the new. This means that the old calculations are closer to the warranty requirements for the solar panels than the new.

Consider an angle λ to the horizontal of incident sunlight to a panel at angle γ to the horizontal, which thus has component $(\lambda + \gamma)$ to the horizontal plane of the panel, and therefore a component $\sin(\lambda + \gamma)$ vertical to it. We wish to maximise this.



If the radiation at the winter solstice is 0.2 units and at the summer solstice 1 unit, as we were using in our calculations in section 2.5.1, and intermediate values are interpolated as a straight line, then we want to maximise

$0.2\sin(15.8° + \gamma) + \sin(62.6° + \gamma)$,

and using the formula

$\sin(A + B) = \cos(A)\sin(B) + \sin(A)\cos(B)$,

we are seeking to maximise

$0.2[\cos(15.8°)\sin(\gamma) + \sin(15.8°)\cos(\gamma)] + [\cos(62.6°)\sin(\gamma) + \sin(62.6°)\cos(\gamma)]$

$= 0.6526 \sin(\gamma) + 0.9423 \cos(\gamma)$,

which we will write as

$C \sin(D + \gamma) = C \cos(D)\sin(\gamma) + C \sin(D)\cos(\gamma)$,

so

$C \cos(D) = 0.6526$
$C \sin(D) = 0.9423$,

giving

$\tan(D) = 1.444$
$D = 55.3°$,

and the maximum of $\sin(55.3° + \gamma)$ is $\sin(90°) = 1$, so

$\gamma = 90° - 55.3° = 34.7°$.

2.7.7.* Characteristics for increasing output by percentages.

* = omitted in the first edition