Enhancing power output using reflectors for solar panels on flat roofs

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Enhancing power output using reflectors for solar panels on flat roofs is to be found as item 3 in the Engineering section of www.jimhadams.com.

The technical part of the report should be accessible to those with a science A level. Its contents are designed for those who wish to educate themselves about the theory and practice of solar panels. Its recommendations may be of interest to decision makers who wish to increase the efficiency of their photovoltaic (PV) installations.

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1. Executive summary.

1.1. Scope and specification.

The present work arose from a specification by Will Cottrell of Brighton Energy Co-op who wanted to install reflectors for fixed photovoltaic (PV) arrays on flat roofs. The questions he asked were

- (A) Is it necessary to cover all of a PV panel with additional radiation from a reflector in order for the output of the panel to be boosted by the increased illumination?
- (B) What is the optimum panel reflector setup on a flat roof?
- (C) Is there a design that can illuminate a panel from below using a reflector, and will this work?
- (D) What is the additional output you expect?

1.2. Summary.

On average about half of the light from the sun which reaches the ground reaches it directly and the other half comes from blue sky or clouds. We find that for a solar panel nearly facing south, a slope angle of 30 degrees boosts output by 10%. When the panel is not near to south facing, a steeper slope, say 35 degrees, is better.

1.3. Recommendations.

2. Technical report.

2.1. Introduction.

A previous work by Jim H. Adams and coworkers "The optimal arrangement of reflectors in solar panel tracker arrays" originated from discussions with Graham Ennis who was in the process of designing PV panels with reflectors. The present report arose from a specification by Will Cottrell of Brighton Energy Co-op, who had looked at that work and the reports of others using reflectors, and wanted to install reflectors for PV arrays on flat roofs.

The purpose of including calculations is twofold

- (i) That computational errors may be detected and challenged.
- (ii) That computations using other assumptions may be more easily developed.

2.1.1. The array and reflector design.

The central new idea in this paper is for a single reflector spanning a spaced array of PV panels, so that rays of light from the sun at different times of day always hit an array from a segment of the reflector that is not necessarily at right angles to a PV panel. This has various consequences.

- (1) There is no interruption to the sunlight on a panel arising from discrete segments of the reflector, which may have negative implications on the output of the panel.
- (2) The absence of reflectors at right angles to the sun's rays at sunrise and sunset means that there is no shadow due to the reflectors under a wide interval of the time of day.
- (3) The only exception to a one reflector, one line of PV array, would be for the back array which could have an additional reflector at a different angle, since these PV arrays are sloped.

2.1.2. Technical summary.

On average about half of the light from the sun which reaches the ground reaches it directly and the other half comes from blue sky or clouds. Using this, we look at what the angle of a south facing solar panel should be. We find output is boosted by 10% for a panel at a 28° slope from the ground compared with a flat panel, for a panel at 35° the output is boosted by 9%, but at 44° by 1%. All these computations are for midday. Outside of midday, and for non south facing panels, the best slope is steeper than 28°.

1.1.3. Technical recommendations.

2.2. Solar radiation.

Solar radiation refers to the electromagnetic radiation that reaches the Earth from the sun. Standard irradiation on a PV panel arises from three sources

- (1) Directly from the sun.
- (2) Indirectly, from radiation scattered in the atmosphere.
- (3) Indirectly, through the medium of clouds.

We will consider the energy and frequency characteristics of illumination arising from these three sources.

The amount of solar radiation on average is as follows [SR1]

Illuminescence type	Percentage
Reflected back into space	35.0
Absorbed by atmosphere	17.5
Scattered to Earth from blue sky	10.5
Scattered to Earth from clouds	14.5
Radiation going directly to Earth's surface	22.5

The outer atmosphere of Earth receives approximately 1367 W/m^2 of solar radiation [World Meteorological Organisation], [SR2]. The radiation varies by around $\pm 2\%$ due to fluctuations in emissions from the sun itself as well as by $\pm 3.5\%$ due to seasonal variations in distance and solar altitude.

By Wien's displacement law, the greater the temperature of a star, the shorter will be the wavelength of its radiant emissions. Solar radiation is spread over a wide frequency range. The sun's rays contain electromagnetic wavelengths as short as 0.2 mm (ultraviolet) with maximum energy centred at around 0.4 mm (visible blue light).

[SPV]. Whilst a cloudy sky can increase the amount of *diffuse* solar radiation, a heavy rain cloud can reduce the *direct* component to almost zero. As there is generally an increase in cloud activity during the colder or wetter months, these factors combine to produce a significant seasonal variation in available solar radiation.

Figure 1 shows that the majority of solar radiation occurs in the short-wave visible and ultraviolet portions of the electromagnetic spectrum. There is a long-wave component of infrared. However, large bands of this are absorbed by gasses and particles within the upper atmosphere. [SR2].

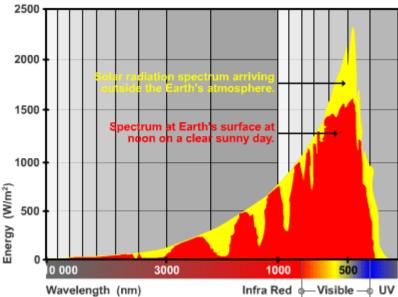


Figure 1 - Spectral content of incident solar radiation.

Ultraviolet (UV) radiation makes up a very small part of the total energy content of solar radiation, roughly 8% – 9%. The visible range, with a wavelength of 0.35 mm to 0.78 mm, represents only 46% – 47% of the total energy received from the sun. The final 45% of the sun's total energy is in the near-infrared range of 0.78 mm to 5 mm. In addition to the spectrum of solar radiation there is a spectrum of terrestrial radiation that fills out the far-infrared range spanning from 3 to 75 mm. These are basically the heat radiating from the surfaces of materials that have been warmed by the sun.

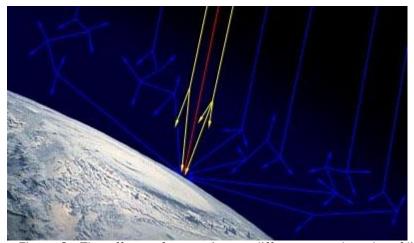


Figure 2 - The effects of scattering on different wavelengths of light. Blue light arrives from all directions after scattering, whilst red and yellow light arrives almost directly from the sun.

You will notice significant differences between the spectral content of the radiation reaching the outer atmosphere and that actually reaching us on the surface. This is due to the absorption of some of the radiation when a gas molecule or particle retains some of this energy as heat. There are noticeable dips in the solar spectrum that coincide with the absorption characteristics of different gasses. Whilst some of this absorbed heat finds its way to the surface as long-wave radiation, the vast majority is simply re-radiated back out into space.

2.3. The response of a PV panel to illumination.

There are three main types of PV panels currently available:

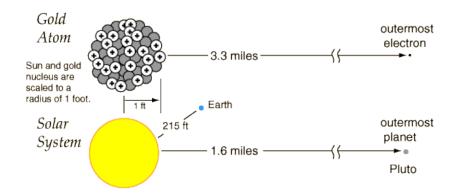
2.3.1. How PV panels work. [Lo1]. [Lo2]

The photoelectric effect, discovered by Edmond Becquerel in 1839, is the mechanism used in PV panels. Direct current electricity (DC) is generated. Some advantages are: zero fuel costs and the free availability of solar fuel, there are no moving parts so there is no noise, reduction in maintenance, the modular design which can also be integrated into buildings and that there are zero emissions. Some disadvantages are: decreasing but high capital costs, at present 20% – 35% efficiency, the use of toxic materials in some manufacturing processes, the need for a power electronic inverter from DC to AC (alternating current) for grid-connected applications, intermittent and variable power and that cloud cover is difficult to predict.

monocrystalline polycrystalline thin film

PV cells under an electric field produce electricity when light particles called photons illuminate a pn junction, to be described. pn junctions are common in semiconductors.

Elements consist of atoms, with a central nucleus containing uncharged neutrons and positively charged protons. Negatively charged electrons lie outside, and in an electrically neutral atom are equal in number to its protons. Here is a relative scale model.



On this scale, the nearest star would be a little over 16,000 Km away. [HP]

Numbers come in two forms: discrete, whole or natural numbers, such as $1, 2, 3 \dots$ and continuous numbers called real numbers, which can include these or lie in any intermediate state between them. Natural numbers may be constrained to loop back to themselves, so they form a finite set of states, such as $0, 1, \dots (n-1)$ with n = 0.

For quantum theory some finite numbers, where these can be called *quantum numbers*, define physical states. For instance these can be finite values representing the angular momentum of a particle. We also have continuous numbers describing physical states, so that whereas the velocity of a photon of light is $\pm c$, particles with rest mass can have any velocities between but not including $\pm c$.

Conventionally, each electron in an atom is described by an *orbital energy state* with four different quantum numbers. The first three (n, l, m_l) specify the particular orbital of interest, and the fourth (m_s) specifies how many electrons can occupy that orbital.

1. Principal quantum number (n): $n = 1, 2, 3, ..., \infty$

Specifies the *energy* of an electron and the *size* of the orbital (the distance from the nucleus of the peak in a radial probability distribution). All orbitals that have the same value of n are said to be in the same *shell* (*level*). For a hydrogen atom with n = 1, the electron is in its *ground state*; if the electron is in the n = 2 orbital, it is in an *excited state*. The total number of orbitals for a given n value is n^2 .

2. Angular momentum (secondary, azimuthal) quantum number (*l*): l = 0, ..., n-1.

Specifies the *shape* of an orbital with a particular principal quantum number. The secondary quantum number divides the shells into smaller groups of orbitals called *subshells* (*sublevels*). Usually, a letter code is used to identify *I* to avoid confusion with *n*:

$$I = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad ...$$
Letter $s \quad p \quad d \quad f \quad g \quad h \quad ...$

The subshell with n = 2 and l = 1 is the 2p subshell; if n = 3 and l = 0, it is the 3s subshell, and so on. The value of l also has a slight effect on the energy of the subshell; the energy of the subshell increases with l (s).

3. Magnetic quantum number (m_l) : $m_l = -l, ..., 0, ..., +l$.

Specifies the *orientation in space* of an orbital of a given energy (n) and shape (l). This number divides the subshell into individual orbitals which hold the electrons; there are 2l+1 orbitals in each subshell. Thus the s subshell has only one orbital, the p subshell has three orbitals, and so on.

4. Spin quantum number (m_s): $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$.

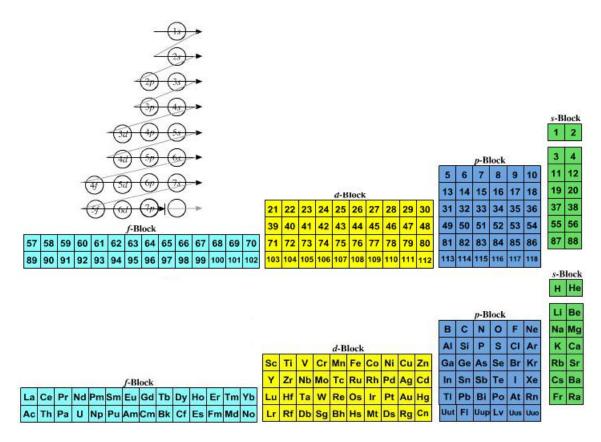
Specifies the *orientation of the spin axis* of an electron. An electron can spin in only one of two directions (sometimes called *up* and *down*).

The **Pauli exclusion principle** states that *no two electrons in the same atom can have identical values for all four of their quantum numbers*. What this means is that no more than *two* electrons can occupy the same orbital, and that two electrons in the same orbital must have *opposite spins*.

TABLE OF ALLOWED QUANTUM NUMBERS

			Number of Orbital Number of		
n	1	m _I	orbitals	name	electrons
1	0	0	1	1 s	2
2	0	0	1	2s	2
	1	-1, 0, +1	3	2p	6
3	0	0	1	3s	2
	1	-1, 0, +1	3	3р	6
	2	-2, -1, 0, +1, +2	5	3d	10
4	0	0	1	4s	2
	1	-1, 0, +1	3	4p	6
	2	-2, -1, 0, +1, +2	5	4 <i>d</i>	10
	3	-3, -2, -1, 0, +1, +2, +3	3 7	4f	14

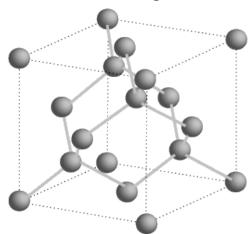
This scheme, or otherwise $2 \times a$ prime number of electrons, generates elements in the **left-step periodic table** shown below. This table is in the same order as the atomic number, which is the number of protons in the nucleus, of these elements.



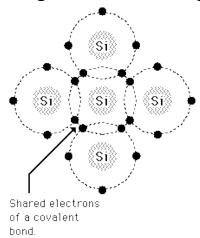
The element silicon, denoted by Si, with atomic number 14 in the above table, is often used in semiconductors. After oxygen, silicon is the most abundant element in the Earth's crust.

Si has 2 electrons in orbital 1s, 2 electrons in orbital 2s and 6 electrons in orbital 2p, leaving 4 outer electrons called *valence electrons*.

Atoms can combine together to form molecules and crystalline structures.

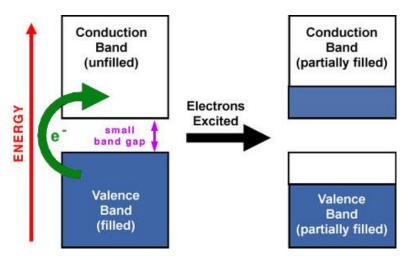


A silicon crystal, a unit cube of which is shown above, forms a diamond lattice. Each Si atom has four electrons which it can share in *covalent bonds* with its neighbours. All valence electrons are tightly held in covalent bonds. A covalent bond, also called a molecular bond, is a chemical bond that involves the sharing of electron pairs between atoms. These electron pairs are known as *shared pairs* or *bonding pairs*, and the stable balance of attractive and repulsive forces between atoms, when they share electrons, is known as covalent bonding. For many molecules, the sharing of electrons allows each atom to attain the equivalent of a full outer shell, corresponding to a stable electronic configuration in a lower energy state than it otherwise would be.



Valence electrons largely dictate the electrical properties of a material.

A useful way to see the difference between conductors, insulators and semiconductors is to plot the available energies for electrons in the materials. Instead of having discrete energies as in the case of free atoms, the available energy states form bands. Crucial to the conduction process is whether or not there are electrons in the conduction band. In insulators the electrons in the valence band are separated by a large gap from the conduction band, in conductors like metals the valence band overlaps the conduction band, and in semiconductors like silicon there is a small enough gap between the valence and conduction bands so that thermal or other excitations can bridge the gap.



Add enough energy to an electron in the valence band of a semiconductor and it "jumps" to the conduction band. A free electron is an electron in the conduction band. In a PV panel a photon excites an electron out of the valence band into the conduction band. Free electrons can flow through the circuit, or in a process known as recombination drop back into the valence band. A built-in electrostatic field pushes the electron through the circuit.

An electron-volt, a unit of energy denoted by eV, is experimentally 1.6×10^{-19} Joules. It is the amount of energy gained (or lost) by the charge of a single electron moving across an electric potential difference of one volt.

By quantum theory, the energy of an electron must fall within well-defined bands. The energy required to jump to the conduction band is known as the *energy gap*. The energy gap is fundamental to the operation of PV panels, varying with the type of semiconductor

Crystalline Si: 1.1 eV Amorphous Si: about 1.75 eV.

Hole: silicon with a missing electron (net positive charge) Hole may attract an electron from a neighbour Process may repeat and hence the hole propagates

- 2.3.2. PV frequency response.
- 2.3.3. Inverters and controllers.
- 2.3.4. Edge of cloud effect.

[EC]. What is the "edge-of-cloud effect" and how can it cause solar array issues?

Clouds are classified by height: high clouds at 5-12 km, such as cirrus, cirrostratus and cirrocumulus, mid clouds at 2-7 km, of type altostratus, altocumulus and nimbostratus and low clouds at up to 2 km, such as cumulus, stratus and cumulonimbus.

"As the cloud begins to cover the sun or when the sun is emerging from behind a cloud, there is a sudden burst of energy that produces more power than normal. This is caused by light refraction. Refraction can concentrate the sunlight while the edge of the shadow passes by. The result is a boost in module voltage output. On a day with bright blue skies and fair weather cumulus clouds, the effect is quite noticeable.

So how can you account for this increase in output? Common practice is to add 20% to 25% to the amperage rating of the solar controller. But many controllers today are the

MPPT type. They track the arrays Maximum Power Point on its IV curve. As the edge of clouds start causing over-irradiance. The MPP voltage starts to rise, so too, does the current. The MPPT controller then adjusts the voltage up to correct for this effect.

Take for example a Sunny Boy 5000 Watt grid-tie inverter. The lower the voltage of the array the better the efficiency. Of course the design of an array depends on the solar panels but you should never design around the highest voltage under standard conditions. In this case 480 VDC. Me, I would design around 350VDC to 400VDC under normal operating conditions. This would allow for the MPP to move around where it wants to".

2.4. The optimal panel slope.

2.4.1. The optimal slope of a panel, by latitude.

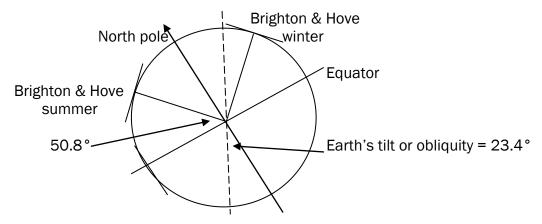
The amount of daily incident solar radiation varies with the time of year and the latitude of the location, where the latitude is measured by the number of degrees north of the Earth's equator. In general, we will denote this angle by θ , where for example Brighton & Hove is at latitude θ = 50.8° N.

We wish to calculate from first principles a good approximation to the slope of a PV array so that the incident solar radiation is at a maximum for a particular latitude averaged over a year.

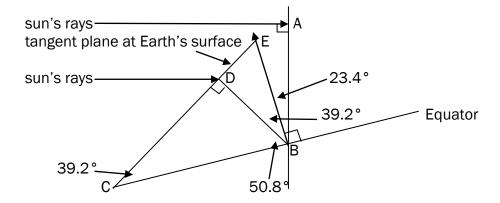
For our calculation we wish to estimate how far from a true value the calculated values at most will be if we adopt a Copernican model of circular motion of the Earth round the sun with uniform velocity and the sun at the centre of this circle.

The distance of the Earth from the sun is very close to 1 astronomical unit, called an AU, which is defined as 149,597,870.7 Km. The nearest approach, called perihelion, is 0.983 AU, and the furthest distance, called aphelion, is 1.017 AU. Their sum, 2 AU, is the longest 'diameter' of the ellipse. Since an ellipse is a squashed circle, we can measure the amount it deviates from a circle by the eccentricity, e, of the ellipse, which for the Earth is 0.0167, so this is very nearly a circle. The focus of the ellipse, which is very near to the centre of the sun, is at a distance given by (perihelion + aphelion) \times e = 0.0334 AU from the midpoint of this 'diameter'. The speed of the Earth at perihelion is 30,300 m/s and at aphelion 29,300 m/s. So we see that if we adopt the simplistic Copernican model, we will get results close to the truth, and we can quantify this.

The rotation axis of the Earth is tilted at an angle, called the obliquity, to the vertical of the Earth's orbital plane around the sun. The obliquity is 23° 26′, where 26′ is 26 minutes or 60ths of a degree, but in decimal notation we can represent this number of degrees as 23.4°.



We will compute the angle of the sun's rays at the summer and winter solstices, on $21^{\rm st}$ June and $21^{\rm st}$ December respectively. At the longest day in summer, the summer solstice, we define this angle in terms of the obliquity and latitude of the array, the latitude chosen being that at Brighton & Hove, $\theta = 50.8^{\circ}$.



The Earth's orbital plane is called the ecliptic. In the diagram above the sun's rays meet the vertical to the plane of the ecliptic at right angles. B is at the centre of the Earth, and in this planar representation the line running through CB is in the plane defined by the equator. Brighton & Hove is represented at D, and the tangent plane at D (the flat roof!) is the line ED continued to C.

The obliquity, angle ABE, is 23.4°. The latitude of D is angle DBC = θ = 50.8°.

Since the angles in a triangle sum to 180° angle BCD + angle DBC + 90° = 180°, which gives angle BCD = 90° - θ = 39.2° .

angle BCD = 90 = 0 = 39.2

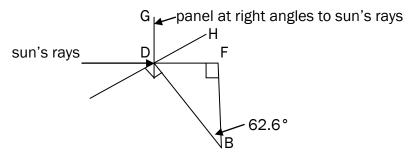
Angle EBC is a right angle, so angle EBD + angle DBC = 90°, or

angle EBD = $90^{\circ} - \theta = 39.2^{\circ}$.

Thus

angle DBA =
$$39.2^{\circ} + 23.4^{\circ} = 113.4^{\circ} - \theta = 62.6^{\circ}$$
.

We now look at more detail around the vicinity of D, Brighton & Hove. We want to find angle GDH, which is the best slope of the PV array with the horizontal under the summer solstice assumptions we have made, assuming these are the most relevant. Please excuse the unrepresentative angles in the diagram!



Since DBF is a right angled triangle angle FBD = 90° - angle FDB, and since angle HDB = 90°, angle HDF = angle HDB - angle FDB

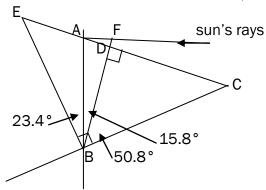
and

we obtain

angle GDH =
$$90^{\circ}$$
 - angle HDF = 90° - 62.6° = θ - 23.4° = 27.4° ,

which is the optimum angle for the panel using this calculation for the sun's rays to be at right angles to the panel at midday of the summer solstice.

We will now look at the situation at the winter solstice, the shortest day of the year.



D is the location of Brighton & Hove and CD is a line tangent to the Earth's surface at D. B again depicts the Earth's centre, and BC represents a line going through the equatorial plane. BE at right angles to BC is the Earth's rotation axis, whereas BA is at right angles to the sun's rays.

We are interested in the triangle FAD, where we will find that angle FAD is the angle at which the sun's rays hit the horizontal, and because the PV panel is assumed to be at right angles to the sun's rays for maximum output, this will give the slope of the PV array with the horizontal.

Angle EBA is the Earth's obliquity, 23.4°. Angle DBC is the magnitude of the latitude of D, $\theta = 50.8^{\circ}$. Since

angle EBA + angle ABD + angle FBC = angle EBC = θ = 90°,

we have

$$23.4^{\circ} + \text{angle ABD} + \theta = 90^{\circ}$$
,

or

angle ABD =
$$66.6^{\circ} - \theta = 15.8^{\circ}$$
.

Now

and triangle FAD contains a right angle, so

giving

$$90^{\circ}$$
 + angle AFD + 66.6° - θ = 180°

angle AFD =
$$90^{\circ}$$
 - 66.6° + θ ,

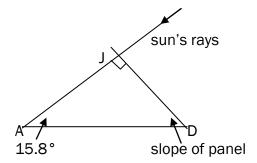
and since angle ADF is a right angle

angle FAD +
$$90^{\circ}$$
 + $(90^{\circ} - 66.6^{\circ} + \theta) = 180^{\circ}$,

which means

angle FAD =
$$66.6^{\circ} - \theta = 15.8^{\circ}$$
.

Thus at the winter solstice, if the panel is at an angle to maximise its output, the slope of the panel, angle JAD in the diagram below, satisfies in the right angled triangle JAD



90° + (66.6° -
$$\theta$$
) + angle JDA = 180°, angle JDA = 23.4° + θ = 74.2°.

We now have the angle for maximum power output due to the sun's rays at the summer and winter solstices, and under the same assumptions, we wish to calculate the maximum power output over the whole year.

We have already mentioned that the effect of cloud is present more in winter than in summer. This will affect the calculation. Also, of course, winter days are shorter than summer ones. Rather than do an explicit calculation to factor in these effects directly, we will take data from *Figure* 3.

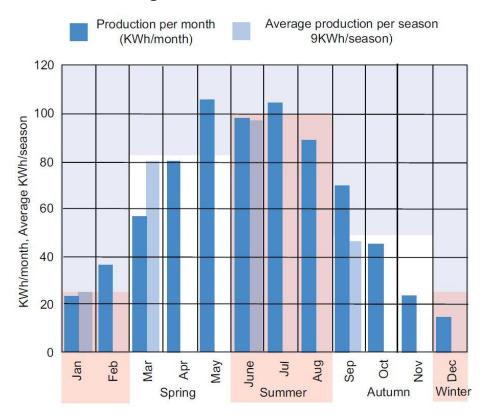
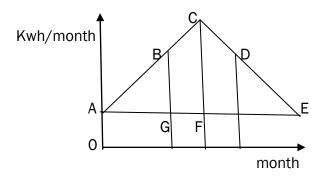


Figure 3. [MCS]. The chart above illustrates the seasonal variation in the energy production of solar panels. I believe that this is based on typical data on how solar panel systems are expected to perform.

From the diagram, we will make the gross approximation that solar energy production is at a maximum of 100 Kwh/month at the summer solstice, and decreases uniformly as a straight line to 20 Kwh/month at the winter solstice.



At which value is the area under AB equal to the area under BC? This should give an approximation to the average month for which the slope of the panel should be maximised. To do this we see that the area of triangle ACF is proportional to $(AC)^2$, and the area of triangle ABG, which has the same angles as triangle ACF, is proportional to $(AB)^2$. We have

½(area ACF + rectangle below it) = (area ABG + rectangle below it).

The month after the winter solstice, m, for which this occurs then satisfies

$$\frac{1}{2}(\frac{1}{2} \times 6 \times 80 + 6 \times 20) = (\frac{1}{2} \times m \times m \times 80/6) + m \times 20$$

giving

$$m^2 + 3m - 27 = 0$$

or by the standard solution of a quadratic equation, and since this solution is positive $m = \frac{1}{2}[-3 + \sqrt{(9 + 4 \times 27)}] = 3.91$ months.

Thus m corresponds to 18th April.

The ratio of m to 6 months is now r = 0.652. We will use this as the factor by which we incline the new slope angle, which is at β_w = 23.4° + θ = 74.2° calculated for maximisation at the winter solstice and β_s = θ - 23.4° = 27.4° calculated for the summer solstice. So we will assume for average maximisation, using the equation of a straight line, it should be at

$$\beta_w + 0.652(\beta_s - \beta_w) = [74.2 - (0.652 \times 46.8)]^\circ = 43.7^\circ.$$

Doly García points out that this assumption might be flaky. We will see later under additional scenarios that whatever reasonable angle we choose gives a not too distant result, so I ignore this.

We have not yet finished. The above calculation only concerns the angle suitable for direct solar radiation. It does not include the effects of scattering of light and light from clouds. We must include these factors in our calculation.

We saw in section 2.2 that these two effects correspond on average to 25% of the radiation from the sun reaching Earth's surface, whereas the effect of direct sunlight was 22.5%. Thus the first effect is 1.1 times that of the second. We will assume that light from indirect sources is on average the same from all points of the sky. For indirect radiation, the optimal panel configuration is horizontal, if we assume that a panel at an angle has its source of radiation restricted by the flat surface, so that it does not come from all angles. Thus at an angle of 43.7° a sector of 43.7/180 = 0.24 of the ambient indirect radiation is blocked out, leaving 76% remaining.

However, we also need to take into account that we are assuming the power output varies as the sine of the incident radiation to the plane of the PV panel. If this angle is γ the normalised power component varies from $\sin(\gamma_1) = \sin(43.7^\circ)$ through $\sin(90^\circ) = 1$ to $\sin(180^\circ) = 0$, and the total summation is the integral

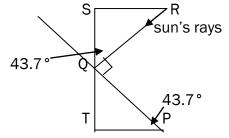
$$\int_{\gamma_1}^{\pi} \sin \gamma \, d\gamma = \, {}^{\pi}_{\gamma_1} [\cos \gamma] = \, 1 \, + \, \cos \gamma_1,$$

where

$$cos(43.7^{\circ}) = 0.723,$$

and we are assuming this integral is invariant over the day. For a horizontal panel the same factor is 2. Thus the power is reduced to $(100 \times 1.723)/2 = 86.15\%$ from indirect radiation.

For the corresponding situation for direct sunlight, we need to know how much the power is reduced by having a flat panel. Since the angle 43.7° is optimum at month m, the direct radiation is at right angles to the PV plane at that month. In the diagram



PQ is the panel. SQ is the vertical component of the sun's rays which are travelling along RQ. If the panel were horizontal, it would occupy part of PT and the total radiation on the panel would be reduced from a factor of 1 to $cos(y_1) = cos(43.7^\circ)$.

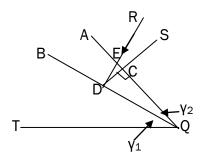
At month m we will compare the radiation on a horizontal panel with a panel at angle γ_1 = 43.7° to the horizontal. We will compare the indirect and direct radiation and their sum.

For a horizontal panel the indirect radiation we have seen is boosted by a factor 1.11 with respect to direct radiation, which has a vertical component $\cos(\gamma_1)$. Their sum is therefore

$$1.11 + \cos(y_1) = 1.11 + 0.72 = 1.83$$
.

For the panel at an angle γ_1 , the indirect radiation we have just seen is $1.11\times(1 + \cos(\gamma_1))/2$, the direct radiation factor is 1, and their sum is $1.56 + [0.555 \times \cos(\gamma_1)] = 1.96$. It can be seen that the increased power at Brighton & Hove on adopting a panel at 43.7° over a horizontal panel is not large. There is a 1% increase.

To find the maximum power output at month m, consider a variation where the angle of the array is incremented from γ_1 = 43.7° by an additional amount γ_2 . We will find the value of γ_2 is negative.



The adjusted panel is now at QA, and the original panel was at QB. Light rays hit QA along the old trajectory RE, and we want to find the vertical component to QA of this along SC. To do this we perform a similar calculation to the ones we have done previously to find that angle EDC is γ_2 . Relative to a length or power value of ED = 1, the value of CD is $\cos(\gamma_2)$. The slope of the adjusted panel is $\gamma_1 + \gamma_2 = 43.7^{\circ} + \gamma_2$.

To find the maximum power output we give a quick course in differential calculus using trigonometric functions like cos and sin. I am assuming the reader has done calculus before.

The derivative, or slope, of a function $f(x) = x^n$ is

$$\frac{df(x)}{dx} = nx^{n-1}.$$

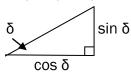
The exponential function is defined as a function whose derivative is given by the function itself. Its value is

$$e^{x} = 1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{3!} + \cdots,$$

where the number n! (called n factorial) = $n \times (n-1) \times (n-2) \times ... \times 1$.

The Pythagoras theorem for a hypotenuse of length 1 can be stated as

$$(\cos \delta)^2 + (\sin \delta)^2 = 1,$$



which we write by an abuse of notation as

$$\cos^2 \delta + \sin^2 \delta = 1$$
.

If we represent the imaginary number

$$i = \sqrt{(-1)}$$
.

S0

$$i^2 = -1$$
,

then we see

$$\cos \delta = \frac{e^{-i\delta} + e^{i\delta}}{2},$$

$$\sin \delta = i \frac{e^{-i\delta} - e^{i\delta}}{2}$$

satisfies the Pythagoras theorem for a hypotenuse of length 1. Expanded out these are

$$\cos \delta = 1 - \frac{\delta^2}{2} + \cdots$$
$$\sin \delta = \delta - \frac{\delta^3}{3!} + \cdots$$

It then follows from the definition of the exponential function that

$$e^{i\delta} = \cos \delta + i \sin \delta$$
.

Thus

$$e^{i(A + B)} = e^{iA}e^{iB}$$

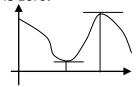
$$= (\cos A + i \sin A)(\cos B + i \sin B) = (\cos(A+B) + i \sin(A+B))$$

so that on comparing real and imaginary parts

$$cos(A + B) = cos A cos B - sin A sin B$$
.

$$sin(A + B) = cos A sin B + sin A cos B$$
.

A local maximum or minimum of a function occurs when the slope or derivative of a function is zero.



From the definition of cos and sin in terms of the series above

$$\frac{\frac{d\sin\delta}{d\delta}}{\frac{d\cos\delta}{d\delta}} = \cos\delta,$$
$$\frac{d\cos\delta}{d\delta} = -\sin\delta.$$

After this brief diversion, we now have the mathematical machinery to find the slope of the PV panel for the maximum panel power output.

For the adjusted panel at an angle $\gamma_1 + \gamma_2$, at month m the indirect radiation we have just seen is

$$1.11 \times (1 + \cos(y_1 + y_2))/2$$
,

with γ_1 = 43.7°, whereas the previous discussion has shown that for direct radiation the value is reduced to the corresponding level

$$cos(y_2)$$
.

The sum of these two values is therefore

$$0.555 + (0.555 \times \cos(\gamma_1 + \gamma_2)) + \cos(\gamma_2),$$

and using the expression for $cos(\gamma_1 + \gamma_2) = cos(43.7^{\circ} + \gamma_2)$,

$$cos(43.7^{\circ} + \gamma_2) = cos(43.7^{\circ}) cos(\gamma_2) - sin(43.7^{\circ}) sin(\gamma_2),$$

or

$$cos(43.7^{\circ} + y_2) = 0.723 cos(y_2) - 0.691 sin(y_2),$$

the sum is

$$0.556 + 1.401 \cos(\gamma_2) - 0.384 \sin(\gamma_2)$$
.

To find the maximum or minimum of this, we have seen we take the derivative with respect to γ_2 and equate it to zero. Thus

$$-1.401 \sin(y_2) - 0.384 \cos(y_2) = 0$$

or

$$tan(\gamma_2) = -0.384/1.401 = -0.274,$$

 $\gamma_2 = -15.3^{\circ},$

but this is a maximum, since if we take its derivative again, it is negative, which means the slope is decreasing as we pass through the maximum. Thus the slope of the panel should be

$$y_1 + y_2 = 28.4^{\circ}$$
.

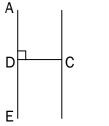
The number we generated for a horizontal panel was 1.83. Using a panel at 28.4° generates a number 0.556 + 0.488 + 0.965 = 2.01, thus the boost in this case is additionally 10% from the horizontal panel case. Note that the computations are for mid day. Outside of midday the effective slope is less.

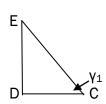
panel from horizontal	0°	28.4°	35°	43.7°
computed boost	0%	10%	9%	1%

2.4.2. Non south facing panels.

At what angle should a PV panel be if not completely south facing?

Let us consider a panel at angle y_1 to the horizontal.



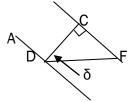


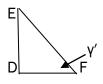
horizontal cross-section

vertical cross-section

The height of the panel is ED, so $\tan \gamma_1 = ED/DC$.

If the panel is now turned by an angle δ ,





horizontal cross-section

vertical cross-section about DF

then in the diagram

DF $\cos \delta = DC$.

If the angle to the horizontal taken through the vertical slice containing DF is γ' , then $\tan \gamma' = ED/DF = DC \tan \gamma_1 \cos \delta/DC = \tan \gamma_1 \cos \delta$.

Now if the sun's rays are parallel to DF, if we are considering a non south facing panel to be sloped at angle γ_1 , we need at Brighton & Hove to specify that the adjusted value of the slope of the panel along DC is

$$v_1 = \tan^{-1}(\tan v'/\cos \delta)$$

to maximise output. Thus the panel should be sloped more.

2.5. Additional output.

2.5.1. Reflector materials and coverings.

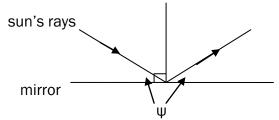
We will assume the mirror film has properties similar to 3M[™] Cool Mirror Film 330.

We can compare this with other, cheaper alternatives. For example, we looked at turkey foil, which has a substantial paper backing. It has 90% reflectivity. The store B&Q supply mirrors.

To protect against birds settling on the reflectors, it is possible to use Tedlar – a plastic foil that resists pigeons. An alternative is Mylar sheet, which is like polythene but very thick. Its transparency is 95%, but it is more expensive.

2.5.2. Reflector characteristics.

A standard result we will use in the diagrams depicting light rays which follow is that the angle of incidence on a mirror equals the angle of reflection, ψ , shown below.

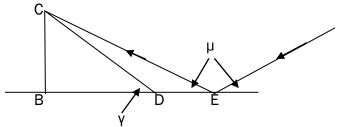


3M™ Cool Mirror Film 330 has an average reflectance at normal incidence of 89% or greater. This reflects light that is useful for PV modules and transmits infrared light. When used in low-x concentrated photovoltaic systems, it can be used to increase the amount of usable light on the module while limiting the amount of heat-generating infrared light on the module. At normal incidence angles, it has low visible reflectance, which limits blinding reflections.

We will look at the effects of the reflector on temperature, and therefore efficiency, of the panel. We will also investigate the frequency of reflected solar radiation in terms of the characteristics of the panel.

2.5.3. The main reflector configuration.

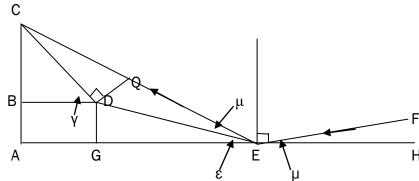
We will first look at the case of direct sunlight on a panel CD at an angle γ to the horizontal where a reflector is placed horizontally next to it.



Suppose the sun's rays are at angle μ to the horizontal. We have seen that the sun's rays are at right angles to a panel at month m when the slope of the panel is 43.7°. This means the sun's rays meet the horizontal at

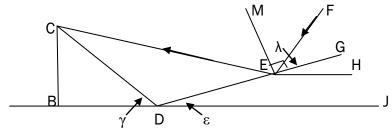
$$\mu = 90^{\circ} - 43.7^{\circ} = 46.3^{\circ}$$
.

For angle γ = 28.4°, this is less than μ , and so for these angles a computation gives the length DE of the flat reflector unphysically as negative! This means we might consider whether the following configuration is possible instead.



The panel at CD is now elevated by a height BA = DG above the flat roof, and the reflector along DE touches the flat roof at E. The edge ray FE continuing to C is now at an angle CEG = μ + ϵ to the horizontal, and since angle μ = 46.3° and angle γ = 28.4°, so γ < μ , this is even worse than the horizontal case previously discussed.

We are therefore led to consider the case when the reflector is at an angle to the horizontal shown below.



The panel CD now adjoins the reflector along DE (or there is a space, for rainwater). The panel angle to the horizontal is γ , the reflector angle is ϵ , EH is a horizontal line and the light ray FE meeting the reflector at E makes an angle of λ with the continuation of DE to DG. Thus

angle EDJ =
$$\varepsilon$$
 = angle GEH,

so since

angle FEG =
$$\lambda$$
 = angle GEH, we have the angle of the light ray to the horizontal angle FEH = μ = λ + ϵ .

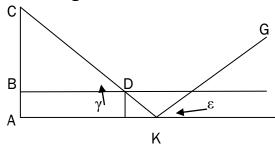
If E is the extremity of the reflector, and a light ray FE is reflected to the end of the panel at C, since DCE is a triangle whose angles sum to 180° ,

angle DCE + angle CED + angle EDC = 180°, angle DCE +
$$\lambda$$
 + (180° - γ - ϵ) = 180°,

SO

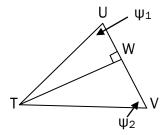
angle DCE =
$$\gamma + \epsilon - \lambda$$
.

We are recommending that to allow rainwater to escape, the panel CD should be elevated to a height BA. Then



an imaginary continuation of CD meets the floor at K and the reflector is now along KG.

We now use the sine rule, whose proof we first provide. Let TUV be the triangle shown below



We have

$$\sin \psi_1 = TW/TU$$
,
 $\sin \psi_2 = TW/TV$,

SO

TW = TU sin
$$\psi_1$$
 = TV sin ψ_2 ,

giving

$$\sin \psi_1/TV = \sin \psi_2/TU$$
.

Consequently

$$\sin (angle DCE)/DE = \sin(angle CED)/CD.$$

Hence

DE = CD sin
$$(\gamma + \epsilon - \lambda)/\sin \lambda$$
.
If $\gamma = 30^\circ$, say, where $\mu = 45.3^\circ = \epsilon + \lambda$, then
DE = CD sin $(30^\circ + 2\epsilon - 46.3^\circ)/\sin(46.3^\circ - \epsilon)$.

Thus a necessary requirement for DE to be positive is that

so

$$46.3^{\circ} > \varepsilon > 8.7^{\circ}$$
,

where ε is the slope of the reflector.

For instance, if we chose $\varepsilon = 30^{\circ}$ then

DE = CD
$$\sin 43.7^{\circ}/\sin 16.3^{\circ} = 2.143 \text{ CD}.$$

However, we have seen at the winter solstice that $\mu = \lambda + \epsilon = 15.8^{\circ}$. If we do not want to block out the sun's rays from the panel at any time of year, then at a minimum value of λ , $\lambda = 0$, we must have ϵ at most as, say, 15°. If we assume this value is the optimum, then we can calculate DE in terms of CD

DE = CD
$$\sin 43.7^{\circ}/\sin 16.3^{\circ} = 2.5 \text{ CD}.$$

It is advised that the value of the length of the reflector be larger than this, so that for smaller values of μ (winter!), the reflector continues to beam over the entire panel. Also panels in a rank in front of the panel should not obstruct the view of the reflector.

We have seen that angle FEH is

$$\epsilon + \lambda = \mu = 46.3^{\circ}$$

so that with $\varepsilon = 15^{\circ}$

$$\lambda = 21.3^{\circ}$$
.

Also we showed the angle of the reflected light ray to the panel is

angle DCE =
$$\gamma + \epsilon - \lambda = 30^{\circ} + 15^{\circ} - 21.3^{\circ} = 23.7^{\circ}$$
,

so the component of reflected sunlight at right angles to the panel is $sin(23.7^{\circ}) = 0.40$.

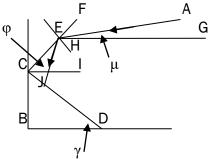
As a second consideration, we will investigate what happens under indirect illumination, where we will consider the radiation for the above ϵ , which now must be the case. Since we have seen that $\mu = \epsilon + \lambda = 15^{\circ} + \lambda$, say, which means indirect radiation varies from 0° to $\lambda = \mu - \epsilon = 30.3^{\circ}$, its proportion of the sky is 30.3/180 = 0.168, or 16.8%.

Relative to the component of direct sunlight at 1, the component of indirect light is 1.11. Thus at the date $m = 18^{th}$ April, the total percentage increase in using reflectors taking account of both direct and indirect illumination is

$$100 \times [1 \times 0.402 + 1.11 \times 0.168]/[1 + 1.11] = 29\%.$$

2.5.4. Illuminating the panel from above.

We will look at a reflector suitable for a back array of arrays in ranks, so that there is no shadow from the reflector for arrays in other ranks, as would be arranged from the diagram



We note that in this configuration, part of the indirect illumination on the panel is blocked off, so we will need to compute this.

The panel is along CD at angle γ to the horizontal, the reflector along CF is at angle ϕ to the horizontal, and the sun's rays, striking the reflector at E are at an angle μ to the horizontal.

```
Now
```

angle ICJ = angle DBC =
$$\gamma$$
 angle ECI = ϕ

SO

angle ECJ = angle ECI + angle ICJ =
$$\varphi$$
 + γ ,

whereas

angle FEG = angle ECI =
$$\varphi$$
,

SO

angle AEF = angle FEG - angle AEG =
$$\varphi$$
 - μ .

What is angle EJC, the angle at which the reflected light ray hits the panel CD? From the property of the interior angles of a triangle

where we have seen

angle JCE =
$$\varphi + \gamma$$
,

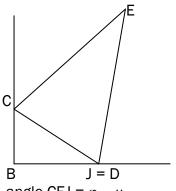
angle CEJ = angle FEA =
$$\varphi - \mu$$
.

Thus

angle EJC =
$$180^{\circ} - \phi - \gamma - \phi + \mu$$

= $180^{\circ} - 2\phi - \gamma + \mu$.

For the length of the corresponding lines, if CJ is extended to CD as in the diagram below



angle CEJ = $\varphi - \mu$,

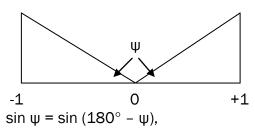
angle EJC =
$$180^{\circ} - 2\phi - \gamma + \mu$$
,

and by the sine rule

so CE, the length of the reflector to cover the whole of the panel CJ = CD, is

CE = CD[
$$\sin(180^{\circ} - 2\phi - \gamma + \mu]/\sin(\phi - \mu)$$
,

and since for an arbitrary angle ψ given in the symmetrical diagram below



we have the length of the reflector

CE = CD[
$$sin(2\phi + \gamma - \mu]/sin(\phi - \mu)$$
.

For example, if CE = CD, then

$$180^{\circ} - 2\phi - \gamma + \mu = \phi - \mu$$
, $3\phi = 180^{\circ} - \gamma + 2\mu$,

or

$$\varphi = 60^{\circ} + (2\mu - \gamma)/3$$
.

At the mid month m = 18th April we are taking μ = 46.3°, so this gives the slope of the reflector

$$\varphi$$
 = 81°.

We recommend ϕ = 82° and extending CE to 2.3 CD, so that the panel continues to be covered by reflected sunlight in summer. For reasons given at the end of this section, it is not desirable that CE be too large, which is why we have increased ϕ by 1°. Under the conditions of the summer solstice we now have

CE = CD
$$\sin(180^{\circ} - 164^{\circ} - 30^{\circ} + 62.6^{\circ})/\sin(82^{\circ} - 62.6^{\circ})$$

= CD $\sin(46.8^{\circ})/\sin(19.4^{\circ})$ = 2.26 CD.

The direct component of solar radiation for ϕ = 81° is now at right angles to the panel by the amount

sin(angle EJC) =
$$sin(180^{\circ} - 2\phi - \gamma + \mu) = sin(18^{\circ} - 30^{\circ} + 46.3^{\circ})$$

= $sin(34.3^{\circ}) = 0.564$.

The indirect component varies over

angle FEG =
$$\varphi$$
 = 81°,

where the total component for unreflected indirect radiation is 180°.

However, for indirect illumination we are also cutting off 180° – ϕ of the angle at C. This means that with the reflector in this position, the indirect radiation covers

$$\varphi - (180^{\circ} - \varphi) = 2\varphi - 180^{\circ} = -18^{\circ},$$

so the reduction of indirect illumination is 18/180 = 0.1.

Thus if we sum the direct components for the reflector at 15° with this top reflector, together with the positive and negative indirect components, we get an increase of

$$100 \times [1 \times (0.402 + 0.592) + 1.11 \times (0.168 - 0.111)]/[1 + 1.11]$$

= $100 \times [0.994 + 0.075]/2.11 = 100 \times 1.069/2.11$
= 50.7% .

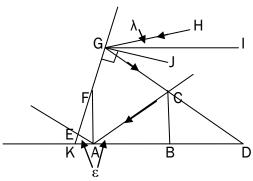
We must bear in mind that the top reflector panels are not available in this configuration to ranks of panels in front of them.

Particles of dust degrade the performance of solar panels. Normal rainwater will wash the panel and thus maintain its effectiveness. The configuration we are discussing will shield part of the panel from rainwater in some circumstances.

Further, the high exposure of the reflector to wind makes it necessary to secure it firmly (no part of the structure to do this should cast a shadow on the panel). There could be a destructive failure of the support structure for the reflector during a gale. It is not recommended that this option is tried unless the structure is stable in a gale, or a movable reflector angle is made to offer minimal wind resistance in such a circumstance.

2.5.5. Illuminating the panel from below.

Why do a calculation on the configuration of panels illuminated from below? The underside of the panel can be illuminated perfectly at midday, but at other times a shadow would be cast due to the struts holding up the panel. This means no additional output would be produced for the time the underside contained partial shadow. However, it is possible to extend the width of the structure on which the panel rests, so its edges meet the floor some distance away from the panel, and this would increase the acceptability of this method summed over a longer period of the day.

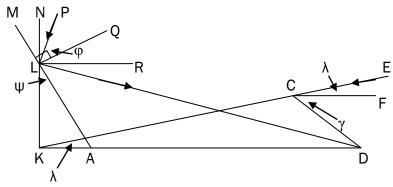


If in the diagram above CD is the panel and EG the reflector, at the winter solstice, a light ray along CA will meet the horizontal AB at an angle ϵ . At the summer solstice, a light ray HG if at angle λ to the horizontal hits the panel after reflection at C.

This arrangement is inconsistent if angle GKA \leq 90°, because λ = 62.6°, so 2λ > angle HGK = 2λ + angle IGJ.

Thus if angle GKA \leq 90°, we must exclude the illumination for part of the year. There are two extremes we might wish to chose: exclude illumination for a period around the summer solstice, or exclude illumination around the winter solstice. However, we have just seen that there can be no illumination below the panel at the summer solstice if angle GKA \leq 90°.

So let us look at the case angle GKA > 90° above, corresponding to angle LAD below.



Let the panel be along CD with slope γ . At the winter solstice a light ray at angle λ = 15.8° goes from C on the panel to K on the flat roof. KN is at right angles to KD. The reflector is aligned along AM at angle ψ to the vertical. KN meets AM at L. QL is at right angles to AL. In general a light ray hits the reflector along PL and meets the base of the panel at D.

angle PLQ = φ = angle QLD.

```
Angle NLR is a right angle. At the winter solstice angle PLR = \lambda, so angle PLD = 2\phi = angle RLD + angle PLR, giving angle RLD = 2\phi – \lambda.
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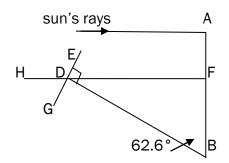
We will fix this value of L as the bottom edge of the reflector. For higher values of λ up to μ = 46.3° at the summer solstice, the reflection will be higher up along the reflector at LM.

2.5.6. Possibly warranty compliant solutions.

The warranty for a PV panel will specify that the conditions under which it operates must not exceed its design characteristics. However, it may be possible to assume that if the panel operates under conditions equivalent to those at the equator, then it does not exceed its warranty requirements. Alternatively, if the PV panel is designed for a certain minimum latitude on the Earth's surface, then under either of these conditions, it might be possible to boost the irradiance of the panel to just below its stipulated maximum, without infringing warranty requirements.

If this condition holds on the warranty, then we can calculate the maximum boost from a reflector to a solar panel which maintains it within the design characteristics for normal operation. Thus it may be of interest to calculate the relative irradiance at the equator and that for the latitude at which the panel is situated.

Say at the equator the incident solar radiation has value 1, and its direct and scattered plus cloud components are in the same ratio as considered previously. Then at the summer solstice for Brighton and Hove at D



angle HDG = 62.6° ,

so this means the vertical component of the sun's rays at D are $sin(62.6^{\circ}) = 0.8878$

so we can boost output and stay within the warranty at that time by at least $100 \times (1 - 0.888) = 11.2\%$.

2.5.7. Maximum output.

2.5.8. Characteristics for increasing output by percentages.

- 2.6. Comparing theory with practice.
- 2.6.1. Review of other information.
- 2.6.2. The practice of PV reflectors on flat roofs.
- 2.6.3. Testing.
- 2.7. Health and safety.

2.7.1. Procedures.

Because of the possibility of blinding sunlight from reflectors, people entering the roof area should wear sunglasses which are not free to fall, or goggles. A danger warning for access to the roof should be prominently displayed, and where to get sunglasses on the premises.

In case of a person not following these instructions, not able to follow them and irrespective of this, training should be provided for emergency procedures when someone is not able to see on the roof, and these procedures should also be prominently displayed. For instance, it could be stated that in the case of blinding reflected sunlight, the person should shut their eyes and cover them with their hands, turn away from the reflection if where it is coming from is known, wait for a minute, and then with fingers moved to see partly, proceed to the exit.

No solar reflectors should be installed where their reflection is visible from the ground or where adjacent buildings may be subject to blinding reflections visible from these buildings at any time of day.

- 2.7.2. The response of the eye to illumination.
- 2.7.3. The effect of reflectors on birds.
- 2.8. Costs.

3. References.

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